



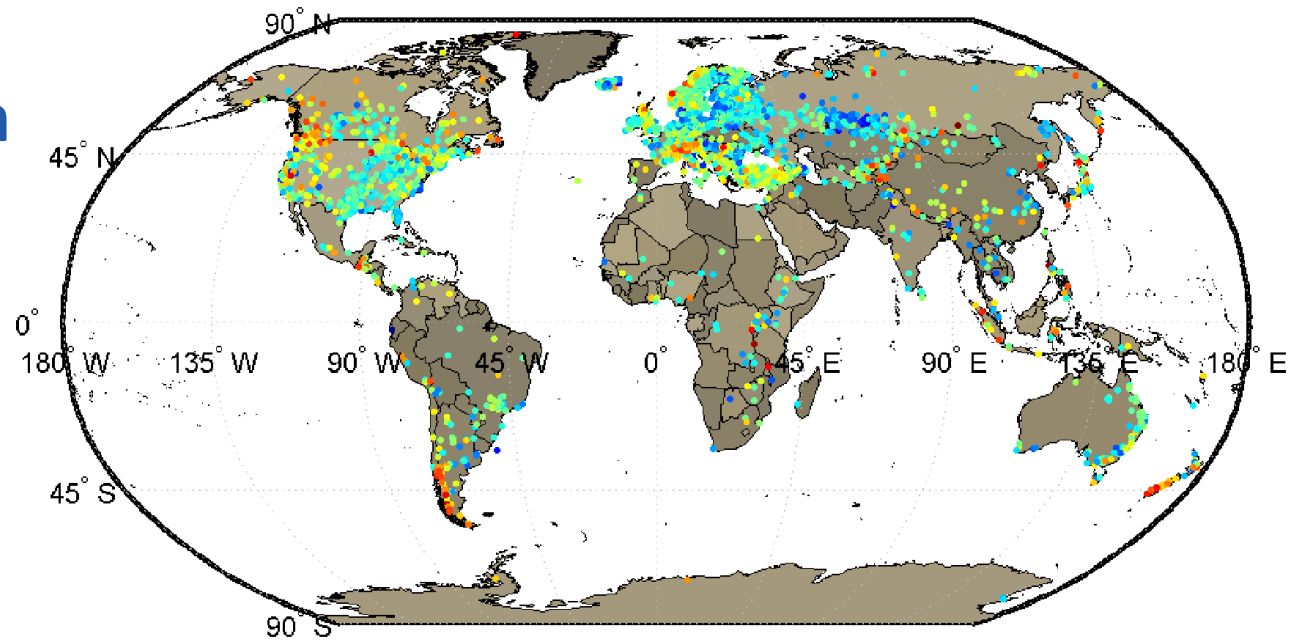
Leibniz-Institute of  
Freshwater Ecology  
and Inland Fisheries

# Perspectives of zero-dimensional modeling for lake representation in climate models

**Georgiy Kirillin and Tom Shatwell**

*5th workshop on parameterization of lakes in NWP and climate modeling*

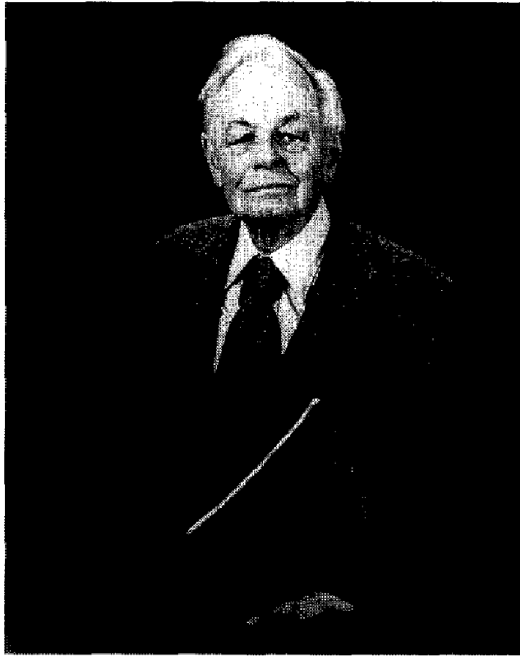
## Motivation



Lake depths from GLDB (After  
Kourzeneva et al. 2012, Choulga  
et al. 2014)

Do we need to model vertical thermal structure of all lakes?

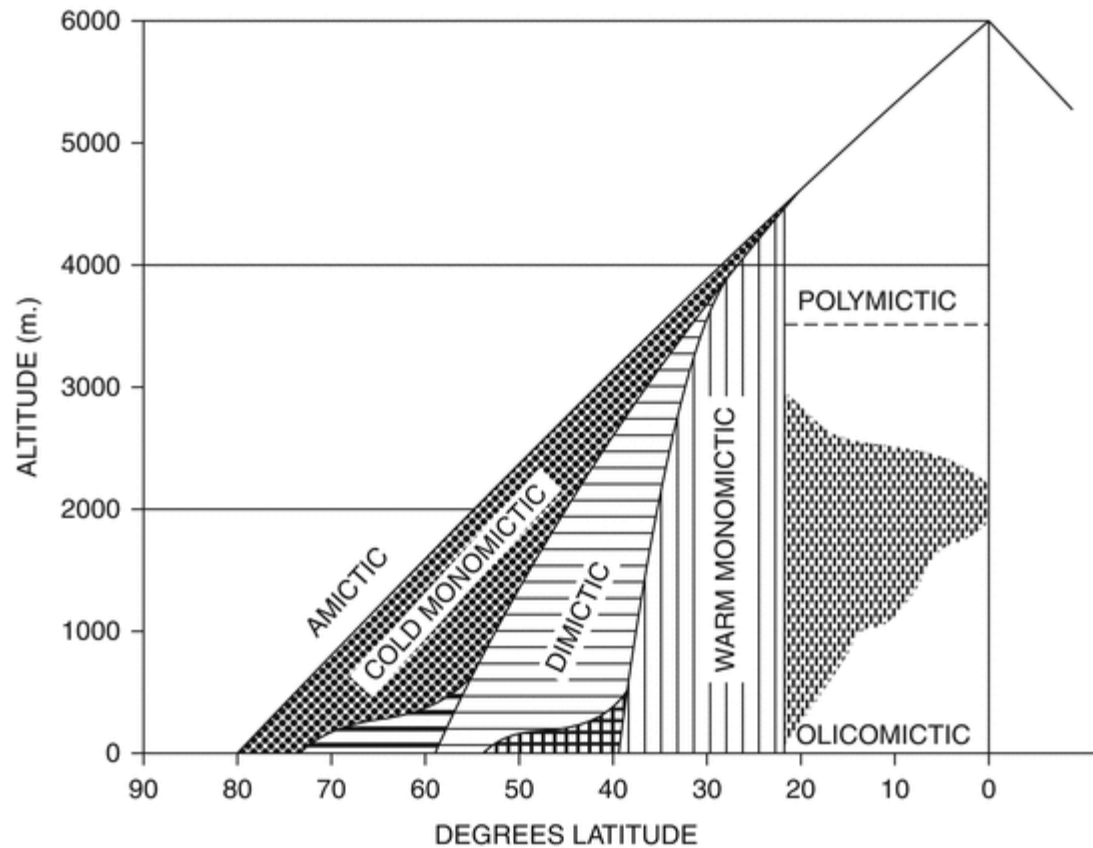
# Motivation



G. Evelyn Hutchinson  
1903-1991

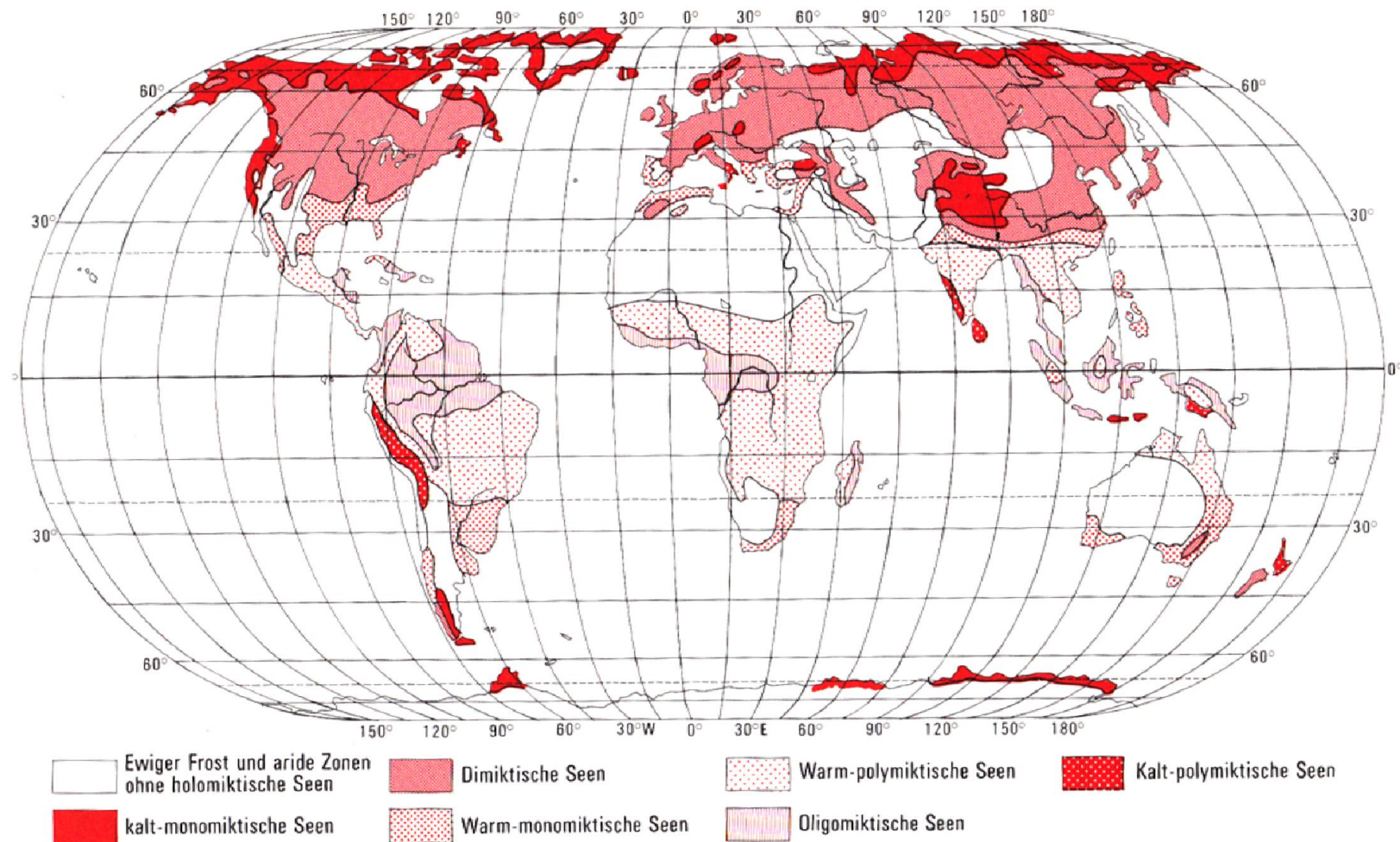
(from Slobodkin and Slack,  
*Endeavour* 23, 1999 )

Hutchinson and Löffler, *PNAS* 42, 1956

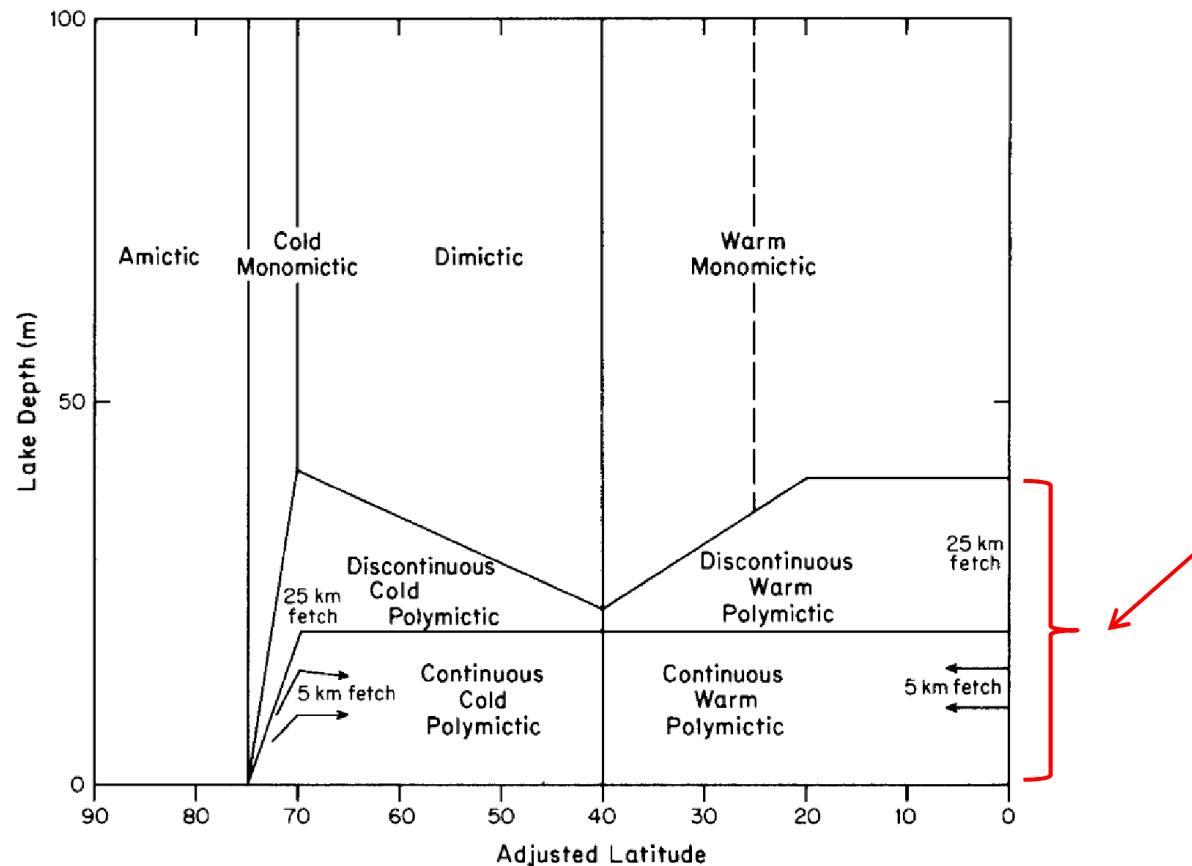


Seasonal mixing regime can be predicted from climatic conditions

# Distribution of lakes according to Hutchinson & Löffler's model



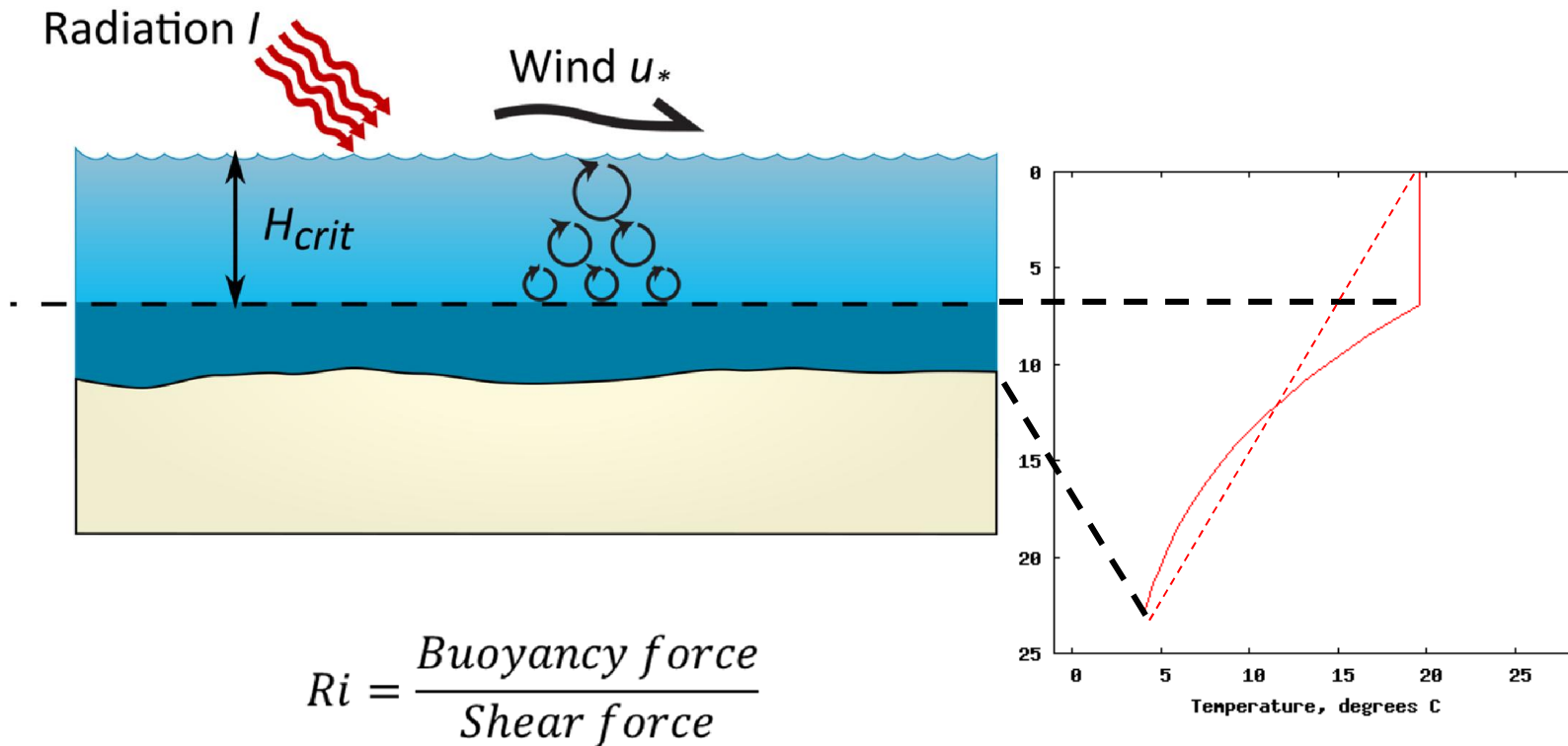
# Extension of Hutchinson&Löffler's model by Lewis (*Can J Fish Aq Sci* 40, 1983)



Lake depth is another important factor for seasonal mixing regime

An attempt to involve fetch into the game

# Richardson Number

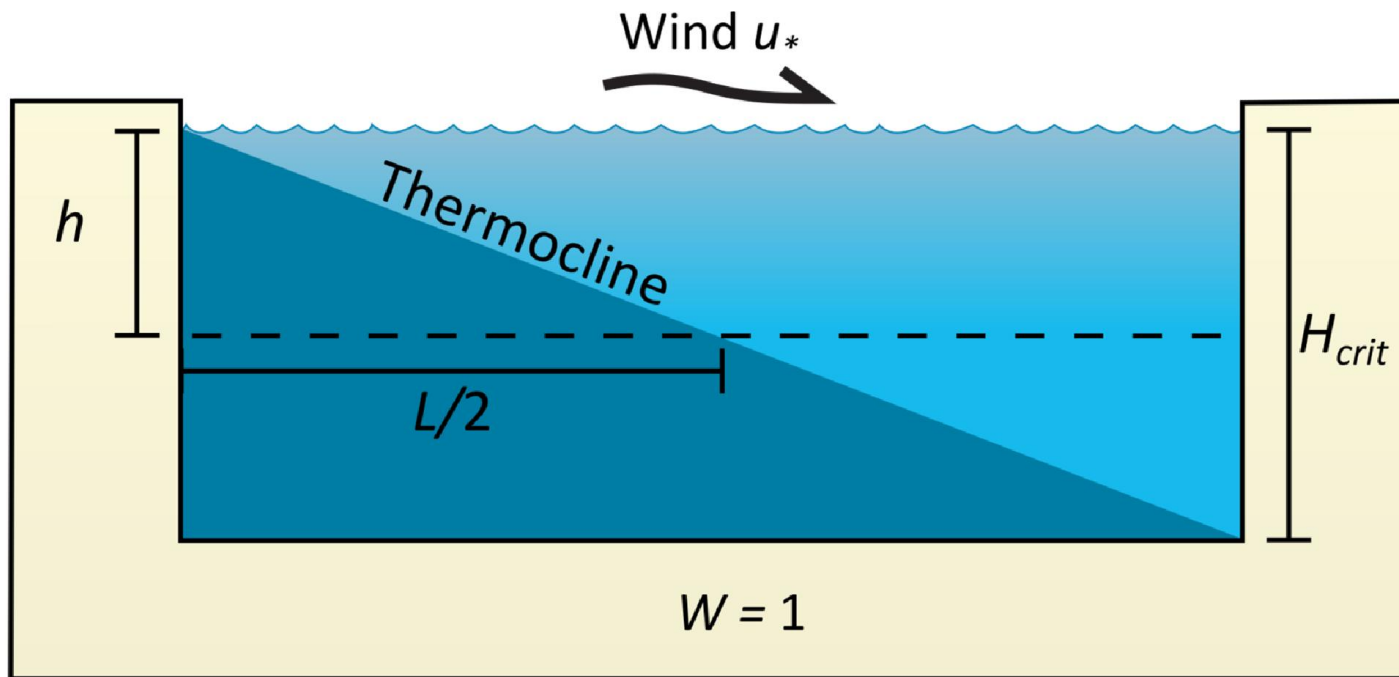


$$Ri_g = (-\partial b \partial z^{-1}) (\partial u \partial z^{-1})^{-2} = N^2 S^{-2}$$

$$Rf = B u_*^{-2} S^{-1}$$

$$Ri_{B*} = \Delta b_0 h u_*^{-2}$$

## Wedderburn Number

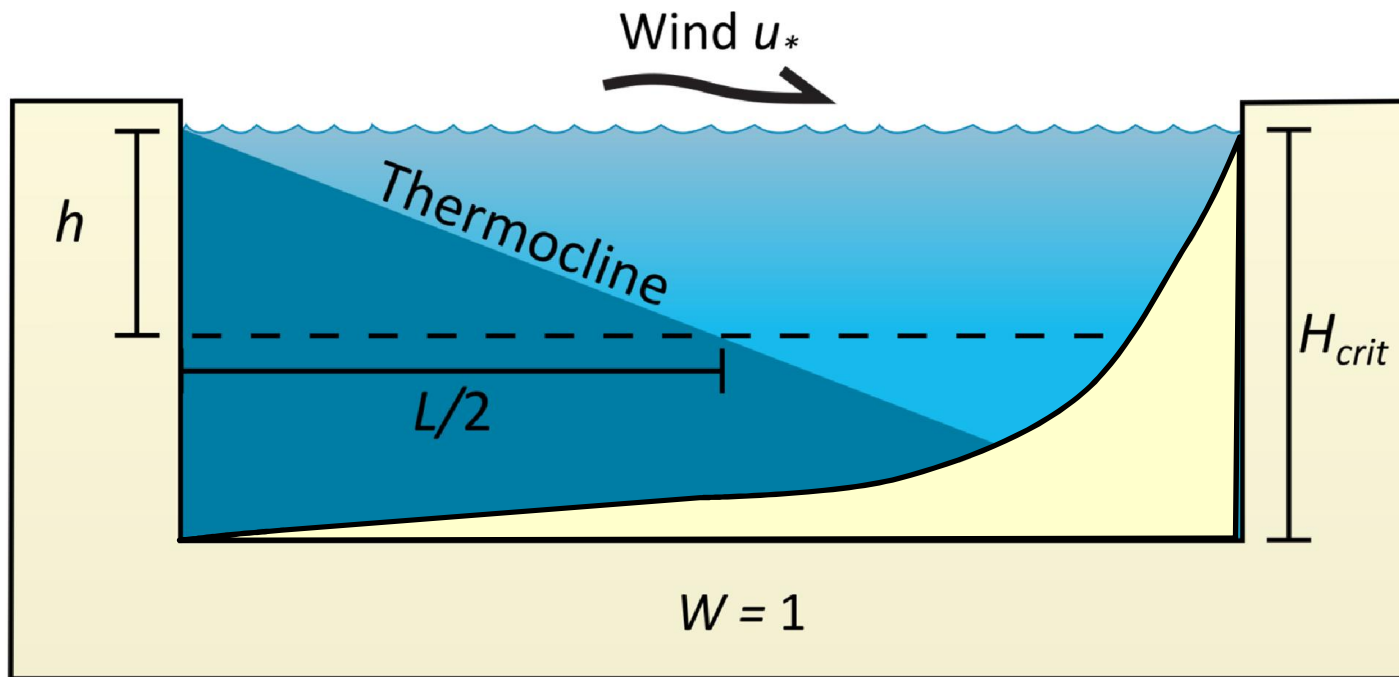


$$Ri_{B*} = \Delta b_0 h u_*^{-2}$$

$$W = \Delta b_0 h^2 u_*^{-2} (L/2)^{-1}$$

$$W = Ri_{B*} A_{1/2}$$

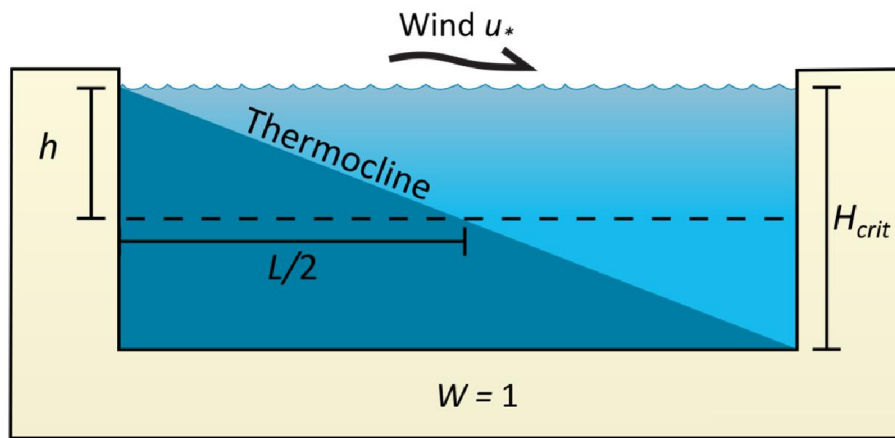
## Lake Number



$$L_N = Ri_s A_{1/2} = \frac{2St h}{L \rho_0 u_*^2 z_v}.$$

$$Ri_s = St (\rho_0 z_v u_*^2)^{-1}$$



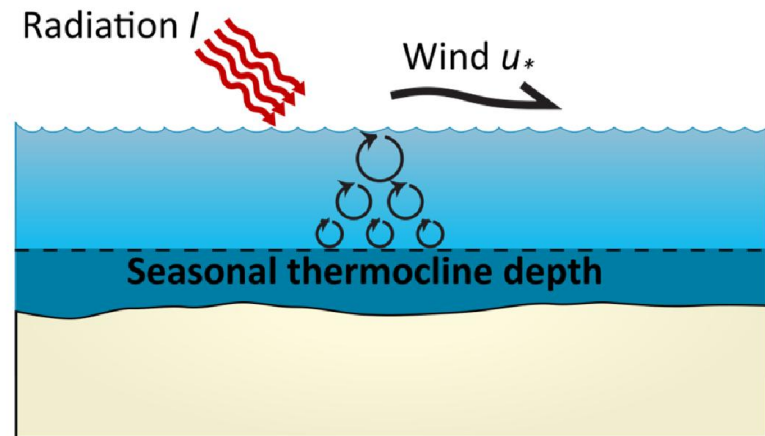


$$W = Ri_{B*} A_{1/2}$$

$W$  is based on *existing stratification* (bulk  $Ri$ ).

To *predict* stratification we need a balance of external forces:

“*Seasonal Richardson Number*”



$$Ri = \frac{\text{Buoyancy force}}{\text{Shear force}}$$

$$Rf = B h u_*^{-3} = w_*^3 u_*^{-3} = h L_{MO}^{-1}$$

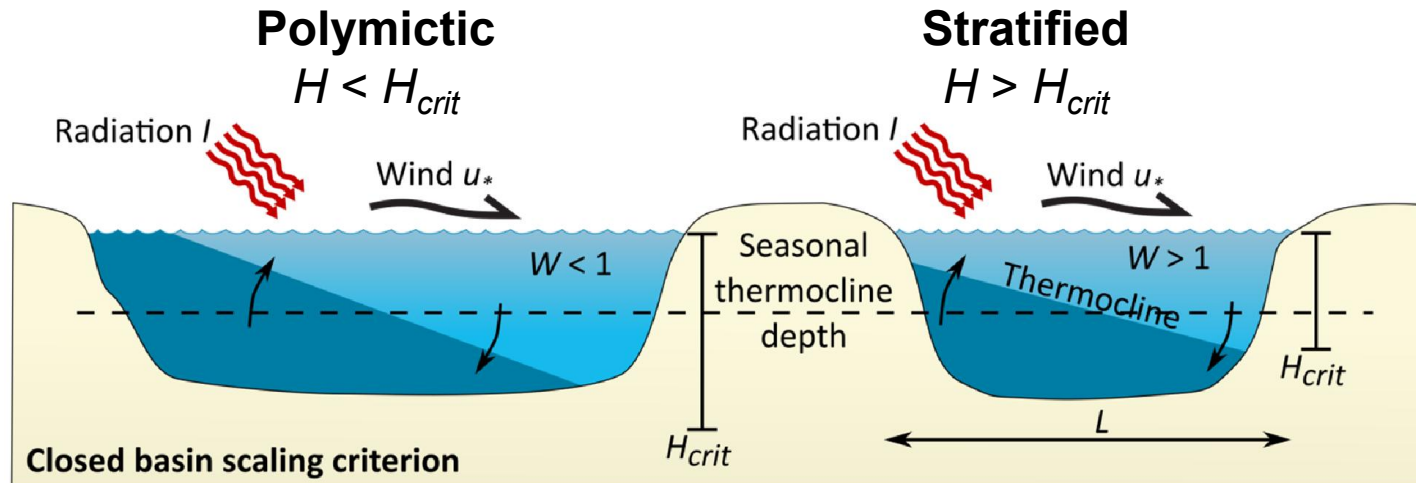
$$B_m = \frac{B_s + B_h}{2} + \frac{1}{2} \left( J_s + J_h - \frac{2}{h} \int_0^h J(z) dz \right)$$

$$J = g\alpha l$$

$$\bar{L}_{MO} = u_*^3 (J_s)^{-1}$$

Further assumptions:  $J(z) = J_s \exp(-\gamma z)$      $h_{SECCHI} \approx 2\gamma^{-1}$

# Generalised scaling of stratification



- Scaling criterion estimates the critical lake depth ( $H_{crit}$ ) so that the seasonal thermocline can tilt to the surface under mean forcing

Scaling criterion:

$$\underline{H_{crit}} = C_1 \underline{h_{SECCHI}} + \sqrt{C_1^2 \underline{h_{SECCHI}}^2 + C_2 \underline{L} \underline{L_{MO}}}$$

Lake aspect ratio

Transparency

Seasonal wind and radiation

## Asymptotic cases

KE budget  
in the Mixed layer:

$$P - hB_m - h\varepsilon_m = 0$$

$$P = C_u u_*^3$$

$$C_u u_*^3 = \frac{J_s}{2} (C_{bs} h - h_{SECCHI})$$

$$\underline{u_*} \gg 0, \underline{B_m} > 0$$

$$h = \sqrt{C_2 L L_{MO}}$$

According to  
Kitaigorodski (1960)  
 $h \sim L_{MO}$

*The difference reflects the  
effect of the lateral  
dimensions  $L$  in lakes.*

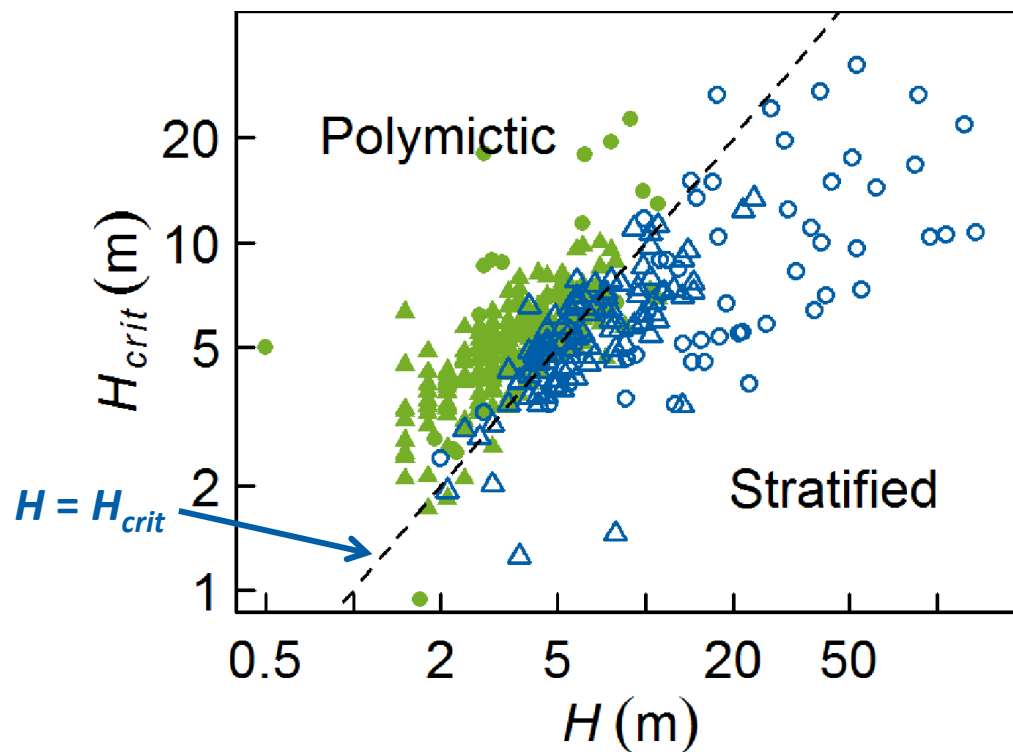
$$\underline{u_*} = 0, \underline{B_m} < 0$$

$$h = h_{SECCHI}$$

*Identical to the result  
of Kraus and Rooth  
(1961)!*

## Validation of scaling criterion

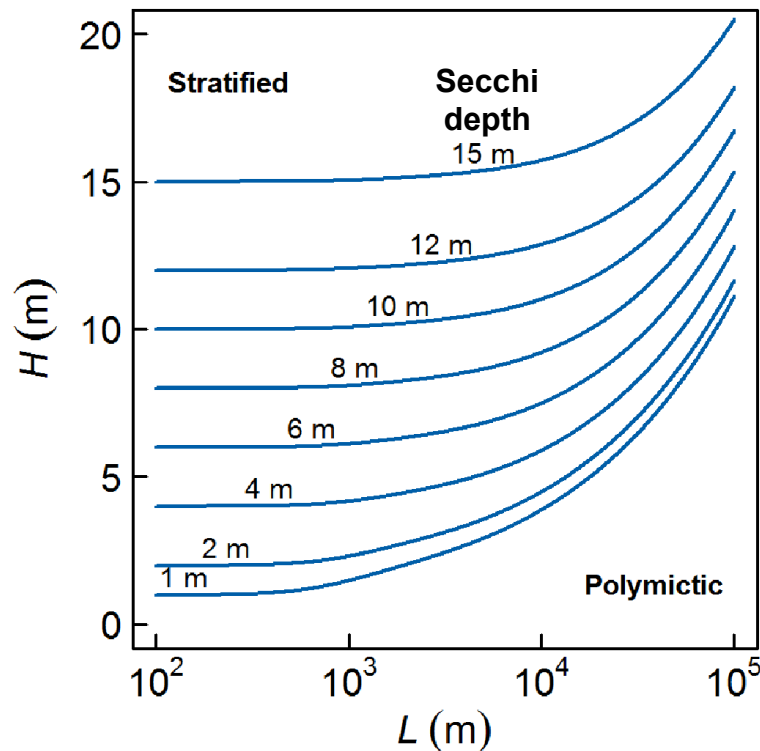
- Fitted to data set of world and American lakes ( $n = 375$ )
- Very good differentiation of mixing regime



$$H_{crit} = C_1 h_{SECCHI} + \sqrt{C_1^2 h_{SECCHI}^2 + C_2 LL_{MO}}$$

Fitted values:  $C_1 = 0.5$ ,  $C_2 = 6 \cdot 10^{-4}$  (0.6 m/km)

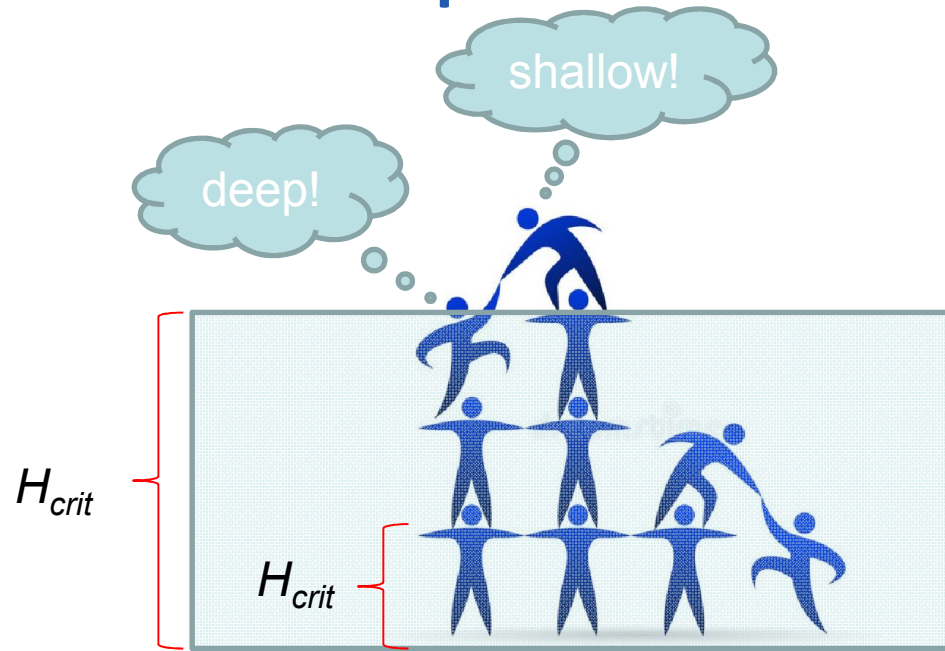
# Effect of transparency and fetch on mixing regime



- In small lakes ( $L < 1000$  m) mixing regime depends on transparency ( $H_{crit} \approx h_{SECCHI}$ )
- In large lakes,  $H_{crit}$  depends on  $LL_{MO}$ , especially wind

$$H_{crit} = C_1 h_{SECCHI} + \sqrt{C_1^2 h_{SECCHI}^2 + C_2 LL_{MO}}$$

# Which lakes are deep?



Our short answer is:

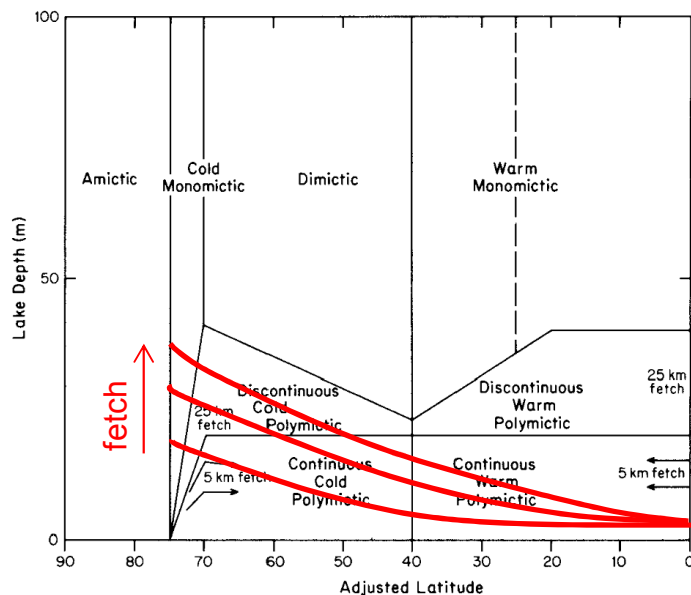
$$H_{crit} = C_1 h_{SECCHI} + \sqrt{C_1^2 h_{SECCHI}^2 + C_2 LL_{MO}}$$

## What to do with this?

$$H_{crit} = C_1 h_{SECCHI} + \sqrt{C_1^2 h_{SECCHI}^2 + C_2 L L_{MO}}$$

Assuming  $L \sim 2$  km,  $h_{SECCHI} = 2$  m:

$$H_{crit} = 1 + (1 + L_{MO})^{1/2}$$

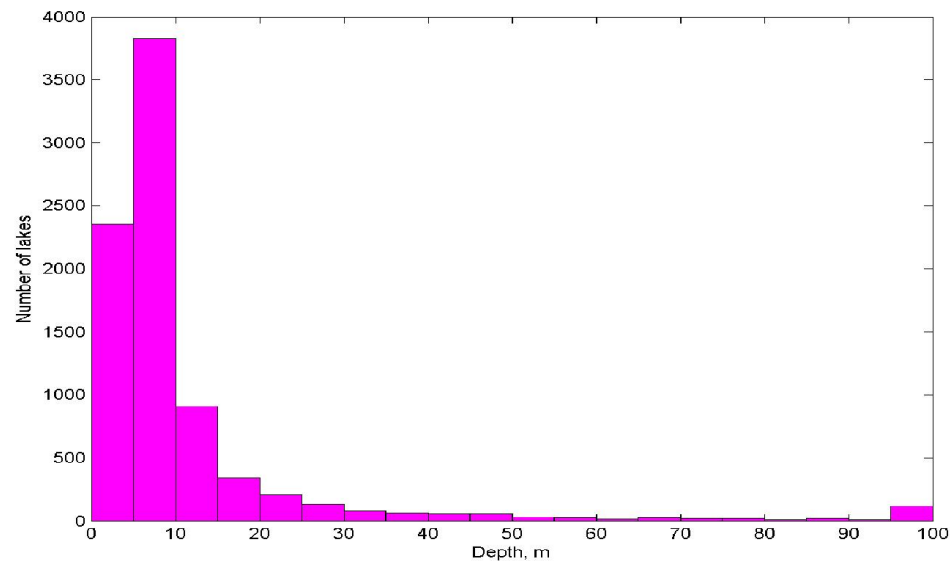


The world lake classification can be refined based on physical criteria

## What to do with this?

$$H_{crit} = C_1 h_{SECCHI} + \sqrt{C_1^2 h_{SECCHI}^2 + C_2 LL_{MO}}$$

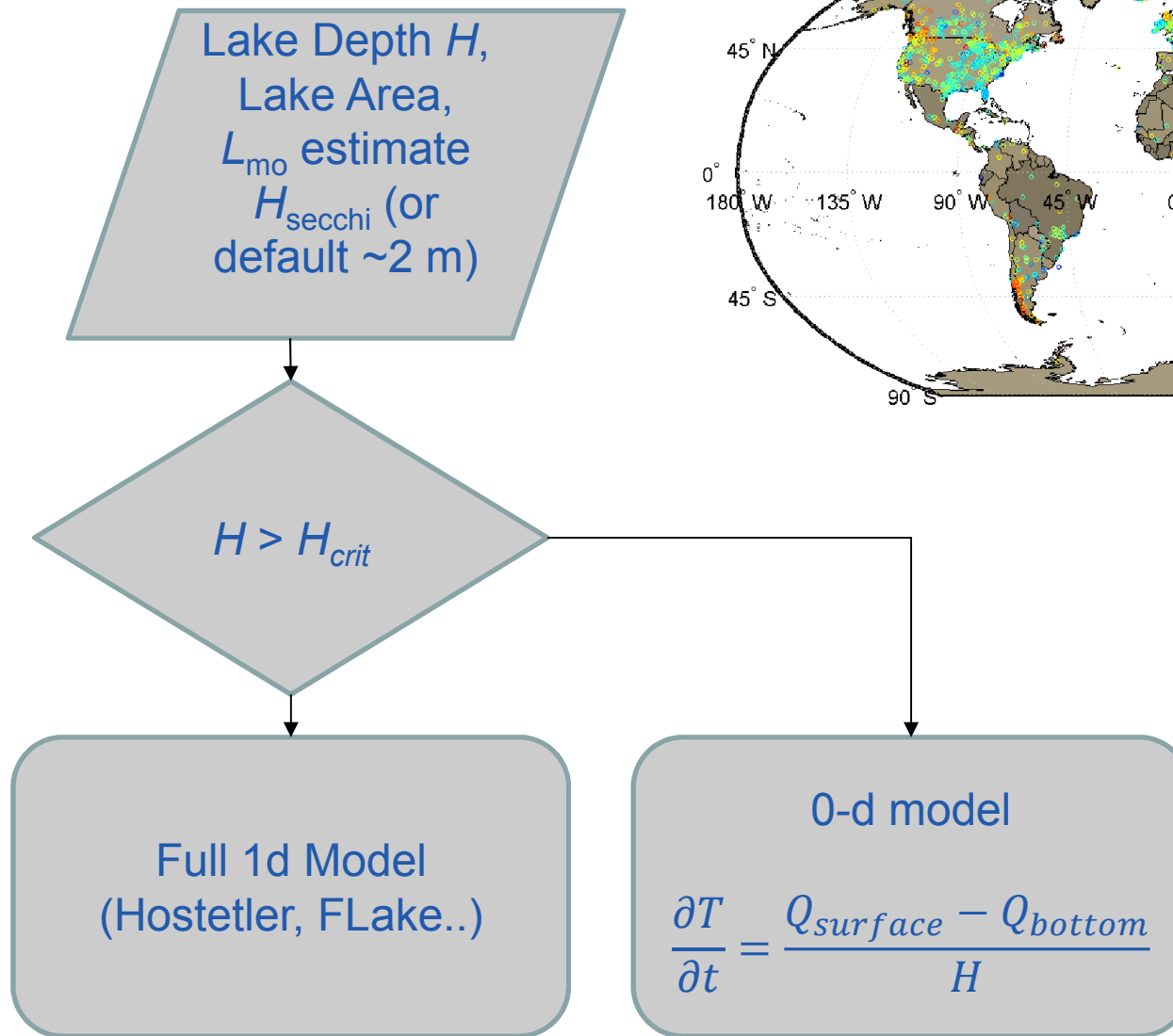
- Climate modeling and NWP:  
avoiding vertical resolution for the majority of lakes in global models
- Upscaling lakes in biogeochemical models



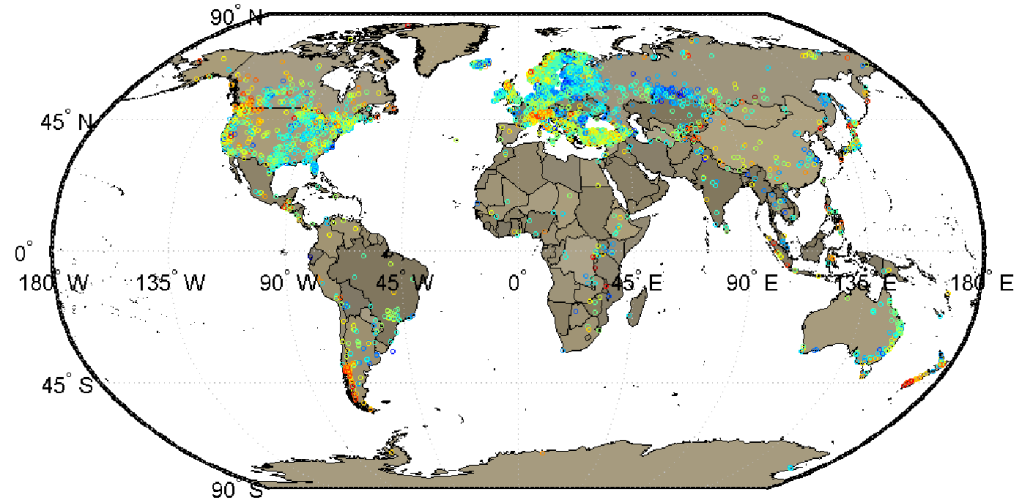
~47% of lakes in the lake depth database are shallower than 10m



# What to do with this?



$$H_{crit} = C_1 h_{SECCHI} + \sqrt{C_1^2 h_{SECCHI}^2 + C_2 L L_{MO}}$$



# Acknowledgements

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Institutions: Smithsonian Environmental Research Center (SERC)

## Thank you for listening

Further reading:

Kirillin, G., and T. Shatwell. "Generalized scaling of seasonal thermal stratification in lakes." *Earth-Science Reviews* 161 (2016): 179-190.



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